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Differential Equations: Calculus AB

Lesson Plan 8: Exponential solutions.

Overview

(As in 6 and 7): This week has a lot of problem-solving, unit-project, and mostly deepening of the knowledge. They have all the tools they need for this unit. Now they need to know how to use those and gain more understanding of these tools.

Learning Objectives

- An application to exponential curves solutions.

Prior Knowledge needed

Separation of variables method for solving. Exponentials and logarithms.

Administration

1. **Unit-Project:** We'll keep on working on it today.
2. ➔ Extra credit 'solve an AP question' on Thursday.
3. Next week presentation and review.

Instruction and activity

1. **Warm-up and review problem (from AP)**.

(Transferred from yesterday: Wanted something short and simple…)

2003 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

   (a) Find $f''(3)$.

   (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

Answer: $y = \left(\frac{x^2+11}{4}\right)^2$.

4. **Growth and Decay solutions (Sec. 6.2 in the book!):**
    a. **No need** to remember new formulas!!
    b. The book states it as a new theorem, but I do not want students to memorize this!
    c. The goal is to understand how it looks like, some properties, and know that it is VERY common.

5. I want to make the following point to remember: take 'absolute' value when solving 'ln' integration. We'll do it through solving $y' = 3y$ with two cases of initial conditions:
    a. $y(0) = 5$ ← solve this with the error in the process, where we do not use absolute-value.
    b. $y(0) = -2$ ← they will then be puzzled about why this doesn't work?!?

6. They have plenty of examples of this in the unit-project, but time permits, do the Newton's law of cooling, page 417, where there are different cases for a rod hotter or cooler then the environment.
7. Important term: 'Half-life'. $y = e^{-t}$ ➔ $t_{\frac{1}{2}} = ln(2)$.
    a. Also: $e^{-1} = 0.37$ $e^{-2} = 0.13$
    b. Always, starting from any point, it is the same half-life.

8. **Unit-project:** continued working.
9. **Wrap-up :** Exponential decay and growth is very common model in real world applications! As time goes, will learn to identify properties of the solution.

10. **Homework:**

    a. Read section 6.2, pay special attention to all the examples.
    b. P.418 (in this order!) : 11-14,25,26,21,24. Choose one of the following sets:
        i. 33->35
        ii. 43,45,47
        iii. 57-58

====End====