zachi@baharav.org
www.baharav.org

Differential Equations: Calculus AB

Unit Project

This document includes all information for the unit-project.

====End====

Unit project – Differential equations

(information page to the student)

* Each group will be responsible to prepare and present a unit-project.

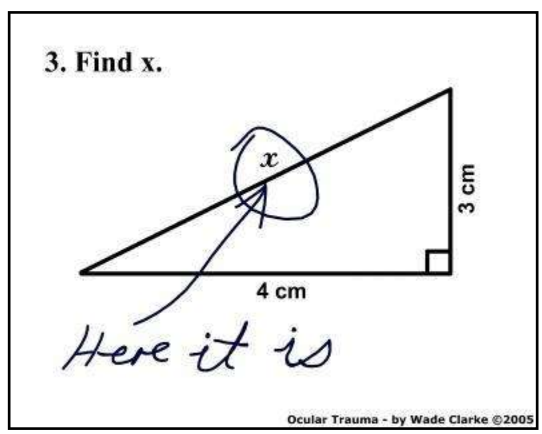
* Each group will be given a differential equation, with possibly some additional details about it, and will need to prepare presentation covering some (or all) of the following aspects:

1. **Equation origin** – Either as a physical model, or word explanation.
    a. **Derivation –** if possible.
2. **Applications**.
3. **History** – Some interesting historical fact(s) related to the equation.
4. **Analytic solution** – Please include solution verification.
5. **Solution curves**.
6. **Slope field**. (can be on the same plot as solution curves).
7. **Special cases** for the solution, and their physical/real-world interpretation.
    a. Initial conditions.
    b. Parameters.
    c. Behavior after a long time.
8. **Possible extensions** of this work.
9. **Other?!?** (be creative)
10. **Last but not the least:** Prepare two questions based on the presentation: One easy and one hard.

* We will start with fact-gathering stage, and toward the end will copy the information on transparencies for presentation.

➔ In the **final presentation**, each member presents at least one slide. The presentation should be between 5 to 10 minutes.

➔ You will need to submit (at time of presentation) a summary of **one-page per slide** that demonstrates the work behind the final representation of this slide. In other words, this is the elaboration/derivation of the slide.



* The audience will be given a form, on which they will need to:

    a. Answer one question related to the presentation.

    b. Supply a constructive feedback.

* If you are running out of ideas on some aspect, come and talk to me: Do not wait to the last minute!!

Have fun, and enjoy the learning experience!

Group 1: RC-Circuit with Voltage Source.

$$RC\frac{dV}{dt} + V = V_s \quad ; \quad V(t=0) = 0$$

V(t) is the unknown function. R (resistor), C (Capacitor) , and V_s (Voltage source) are all known constants.

Group 2: RC-Circuit.

$$RC\frac{dV}{dt} + V = 0 \quad ; \quad V(t=0) = V_0$$

V(t) is the unknown function. R (resistor), C (Capacitor) , and V_0 (Initial Voltage) are all known constants.

Group 3: RL-Circuit with Voltage Source.

$$L\frac{dI}{dt} + RI = V_s \quad ; \quad I(t=0) = 0$$

I(t) is the unknown function. R (resistor), L (Inductor) , and V_s (Voltage source) are all known constants.

Group 4: RL-Circuit.

$$L\frac{dI}{dt} + RI = 0; \quad I(t=0) = I_0$$

I(t) is the unknown function. R (resistor), L (Inductor) , and I_0 (Initial current) are all known constants.

Group 5: Non linear RC Circuit. (Desoer and Kuh, pp. 116-117) (H)

(where $I_R = V_R^3$, and assume C=1F)

$$C\frac{dV}{dt} + V^3 = 0 \quad ; \quad V(t=0) = V_0$$

Group 6: Mass Moving on a Plane with Friction.

$$M\frac{dv}{dt} = -Bv \quad ; \quad v(t=0) = v_0$$

M – known mass ; B – Known friction coefficient ; $v(t)$ – unknown function to be determined.

Group 7: Population Growth Models. (H)

$$\frac{dN}{dt} = kN\left(N_{equi} - N\right) \quad ; \quad N(t=0) = N_0$$

$N(t)$ is the population number, to be solved for. K, N_{equi} , and N_0 are known constants.

Group 8: Radioactive Decay.

$$\frac{dm}{dt} = -km \quad ; \quad m(t=0) = m_0$$

$m(t)$ is the mass to be solved for. k and m_0 are known constants.

Little helper

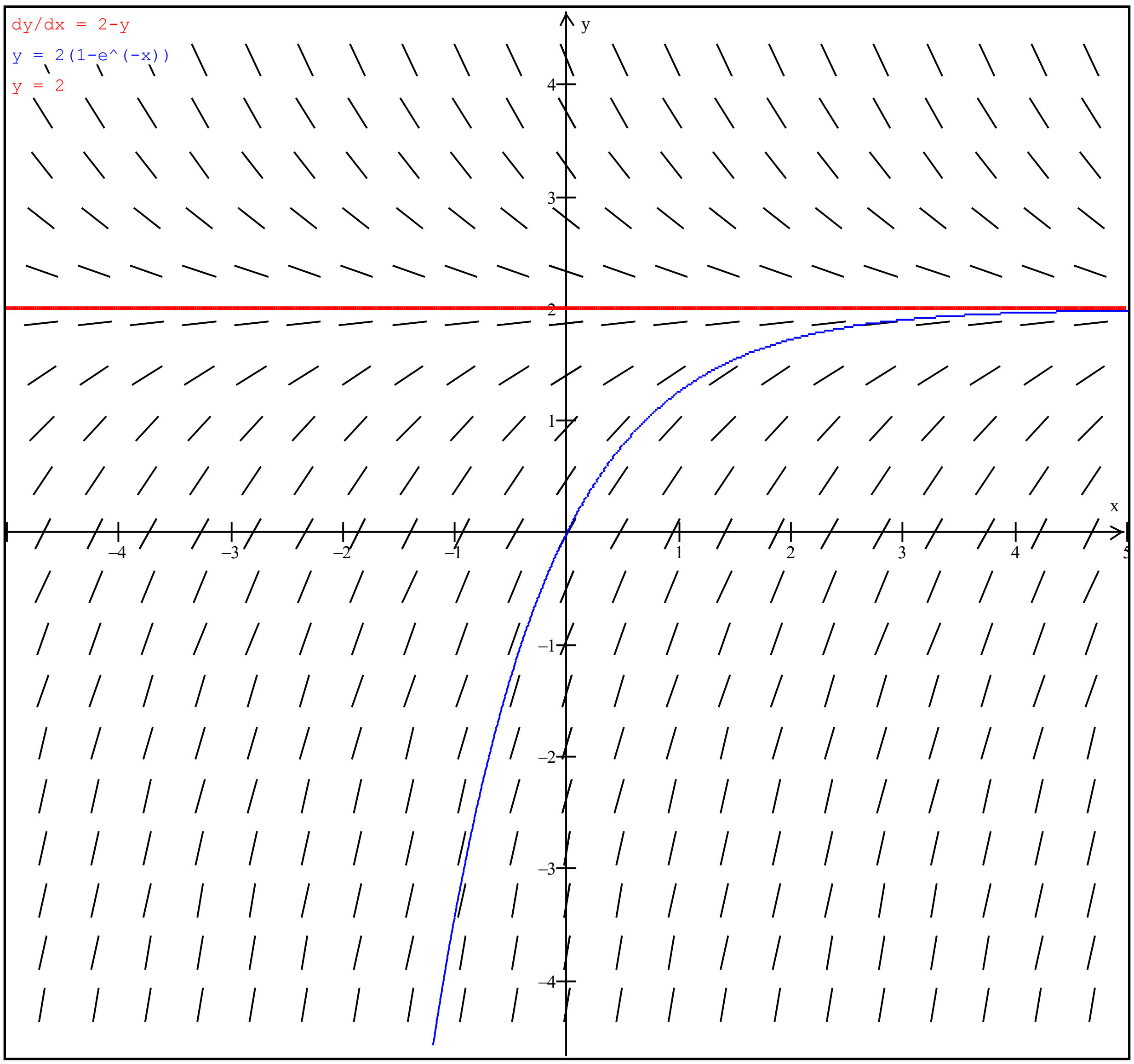
Group 1: RC-Circuit with Voltage Source.

$$RC\frac{dV}{dt} + V \ = V_s \qquad ; \qquad V(t=0) \ = \ 0$$

V(t) is the unknown function. R (resistor), C (Capacitor) , and V_s (Voltage source) are all known constants.

Slope field for R=1, C=1, $V_s = 2$: $$1 * \frac{dV}{dt} + V \ = \ 2$$

Solution curve.



Solve for general R,C,Vs.

What's the asymptote line? Why?

Little helper

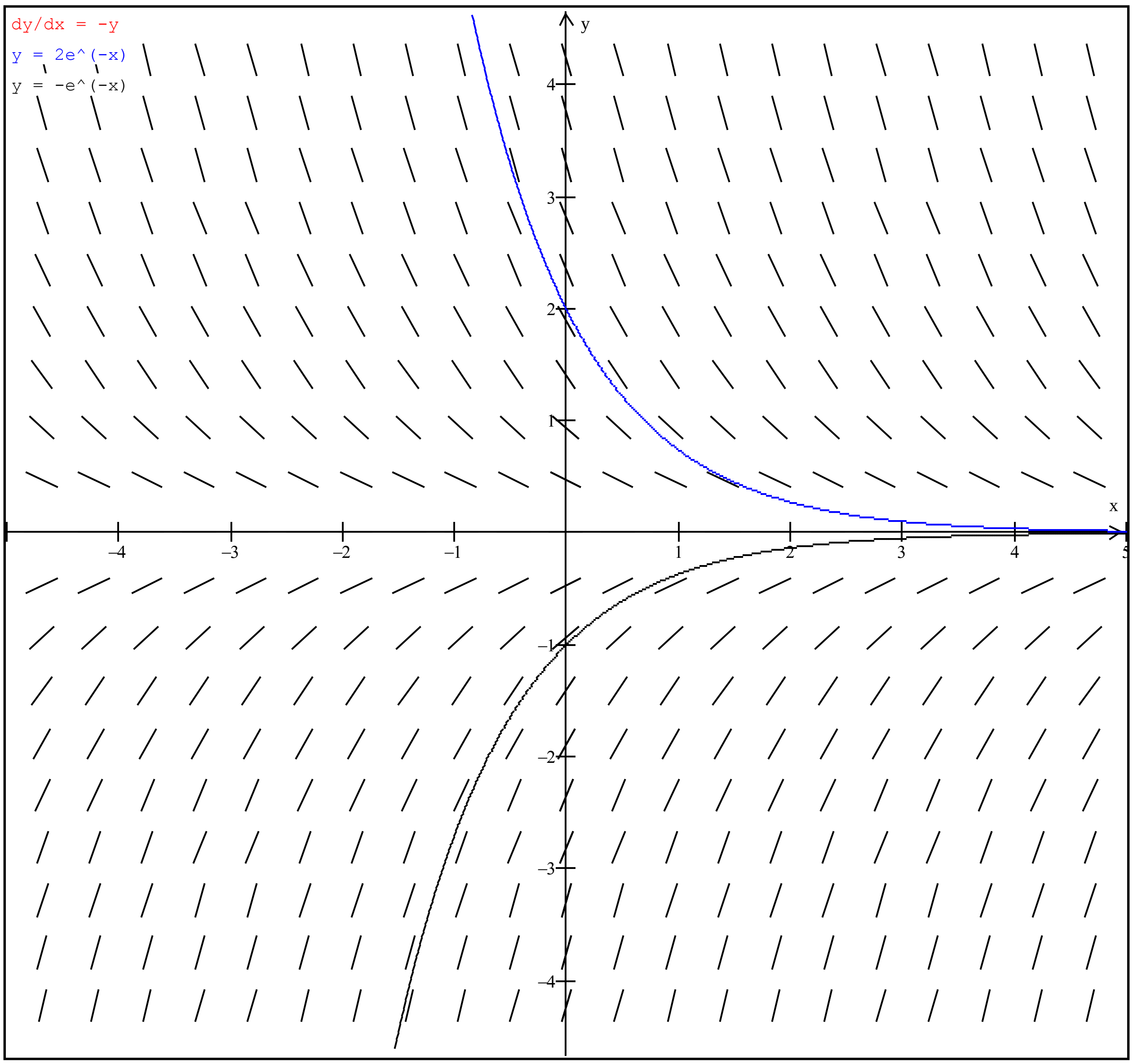
Group 2: RC-Circuit.

$$RC\frac{dV}{dt} + V \;=\; 0 \qquad ; \qquad V(t=0) = V_0$$

V(t) is the unknown function. R (resistor), C (Capacitor) , and V_0 (Initial Voltage) are all known constants.

Slope field for R=1, C=1,: $1 * \frac{dV}{dt} + V \;=\; 0$

Solution curves: Initial condition V(0) = 2 , and V(0)=-1.



Solve for general R,C,V(0).

Exponential decay? Half-value time?

Little helper

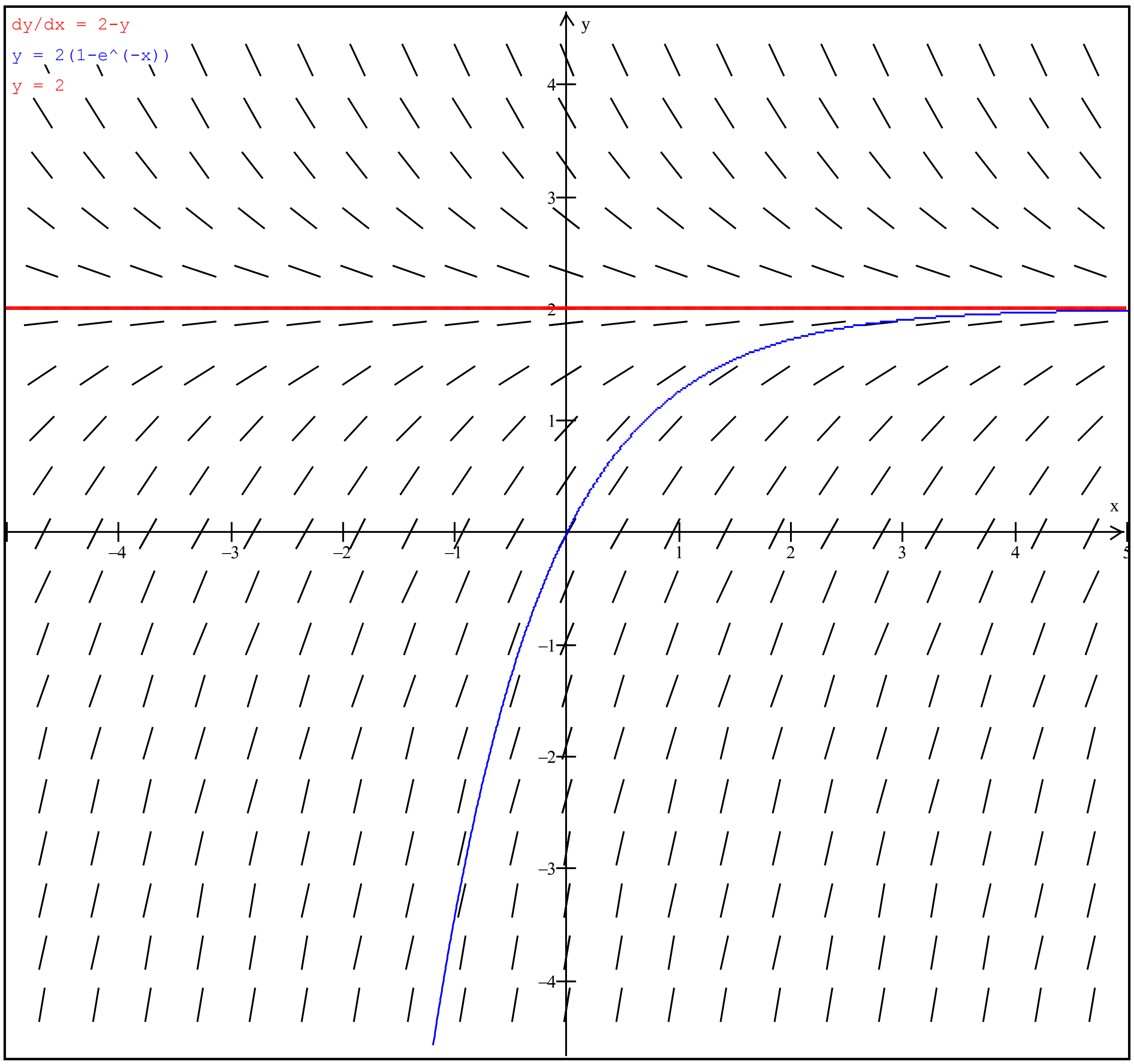
Group 3: RL-Circuit with Voltage Source.

$$L\frac{dI}{dt} + RI = V_s \quad ; \quad I(t=0) = 0$$

I(t) is the unknown function. R (resistor), L (Inductor) , and V_s (Voltage source) are all known constants.

Slope field for R=1, L=1, $V_s = 2$: $1 * \frac{dI}{dt} + I = 2$

Solution curve.



Solve for general R,L,Vs.

What's the asymptote line? Why?

Little helper

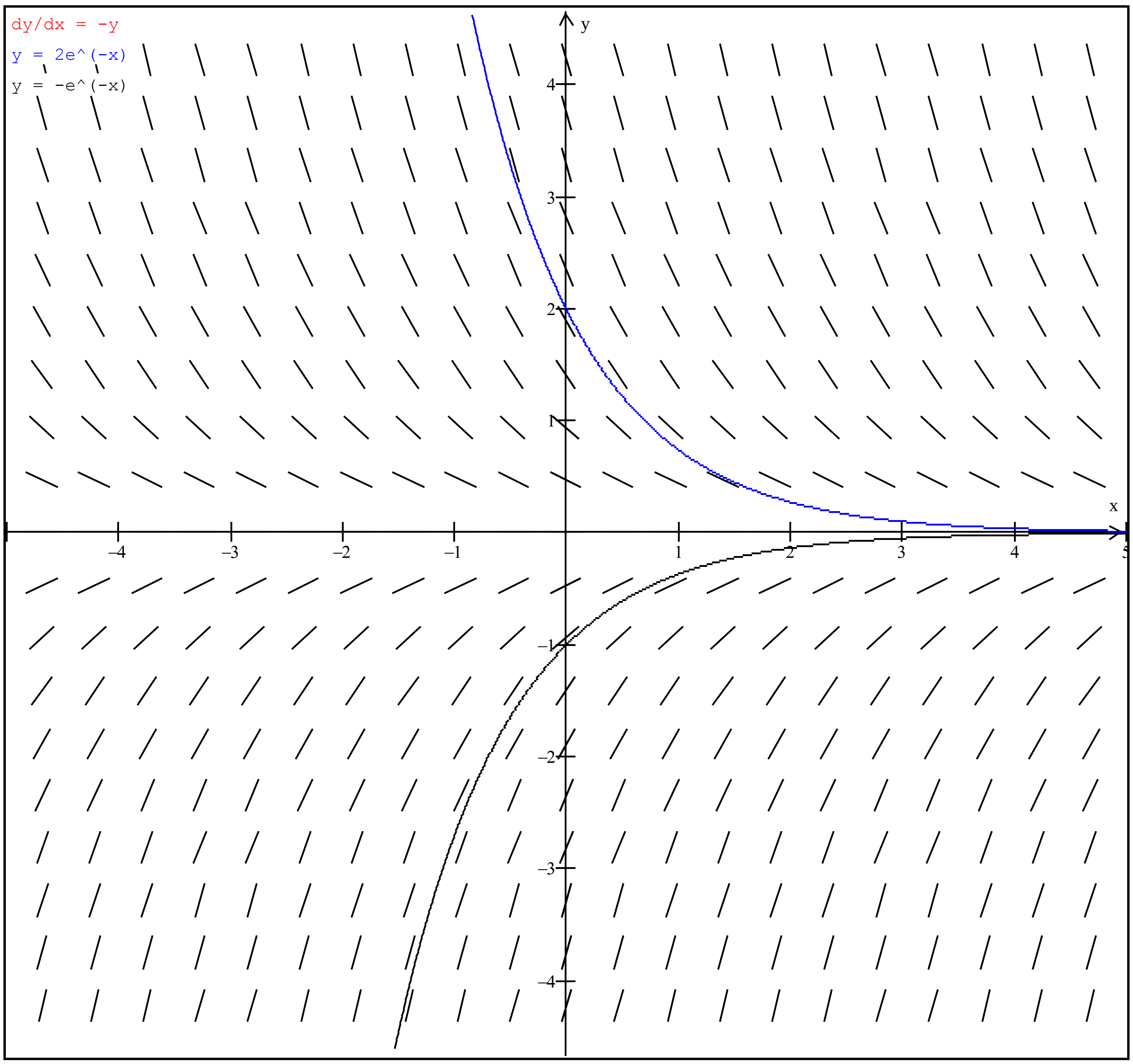
Group 4: RL-Circuit.

$$L\frac{dI}{dt} + RI = 0; \qquad\qquad I(t=0) = I_0$$

I(t) is the unknown function. R (resistor), L (Inductor) , and I_0 (Initial current) are all known constants.

Slope field for R=1, L=1, : $1 * \frac{dI}{dt} + I = 0$

Solution curves: Initial condition I(0) = 2 , and I(0)=-1.



Solve for general R,L,I(0).

Exponential decay? Half-value time?

Little helper

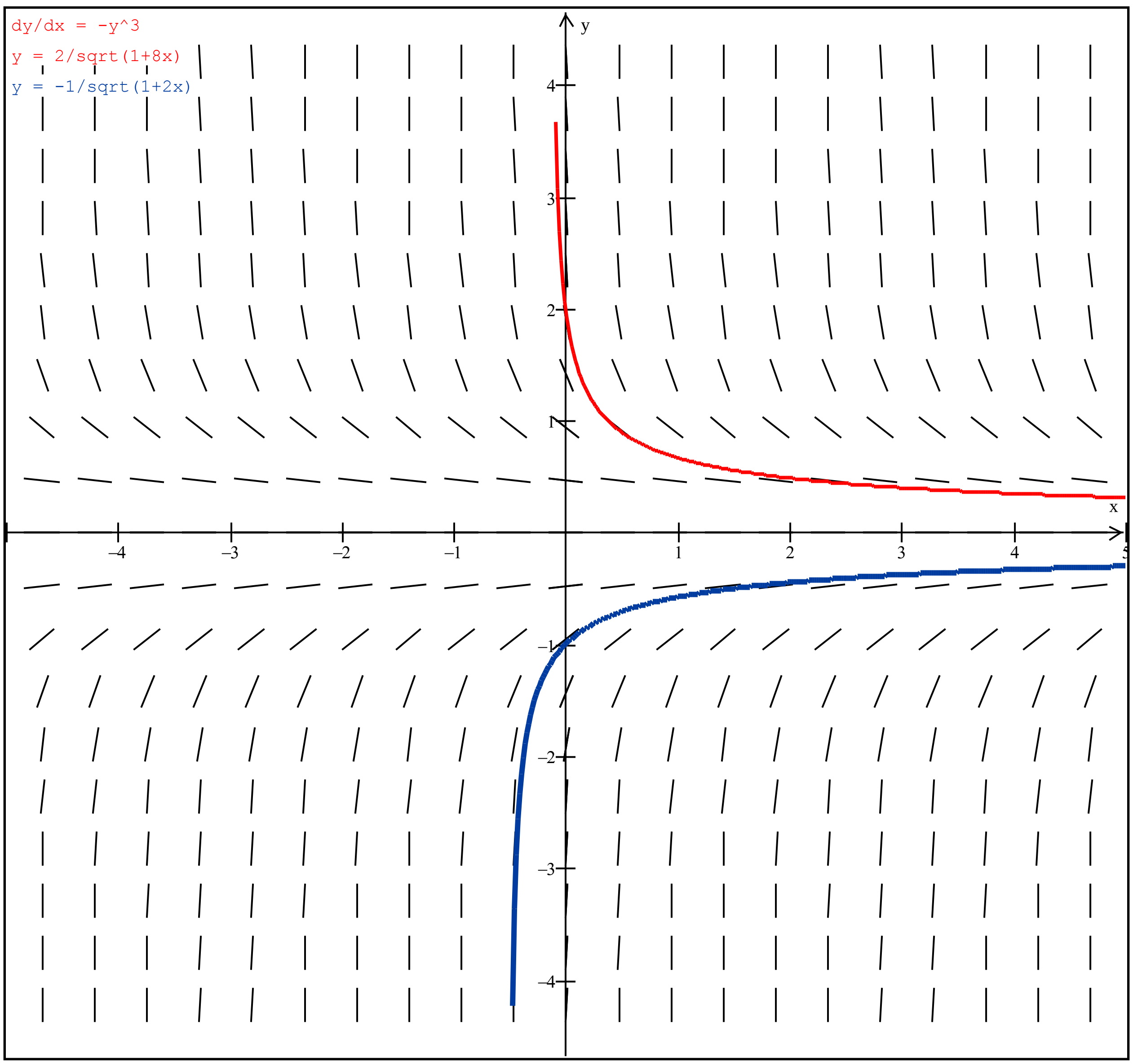
Group 5: Non linear RC Circuit. (Desoer and Kuh, pp. 116-117) (H)

(where $I_R = V_R^3$, and assume C=1F)

$$C\frac{dV}{dt} + V^3 = 0 \qquad ; \qquad V(t=0) = V_0$$

Slope curve for C=1 : $1 * \frac{dV}{dt} + V^3 = 0$

Solution curve for : $V_0 = 2$ and $V_0 = -1$



Find the general solution in terms of Vo.

Little helper

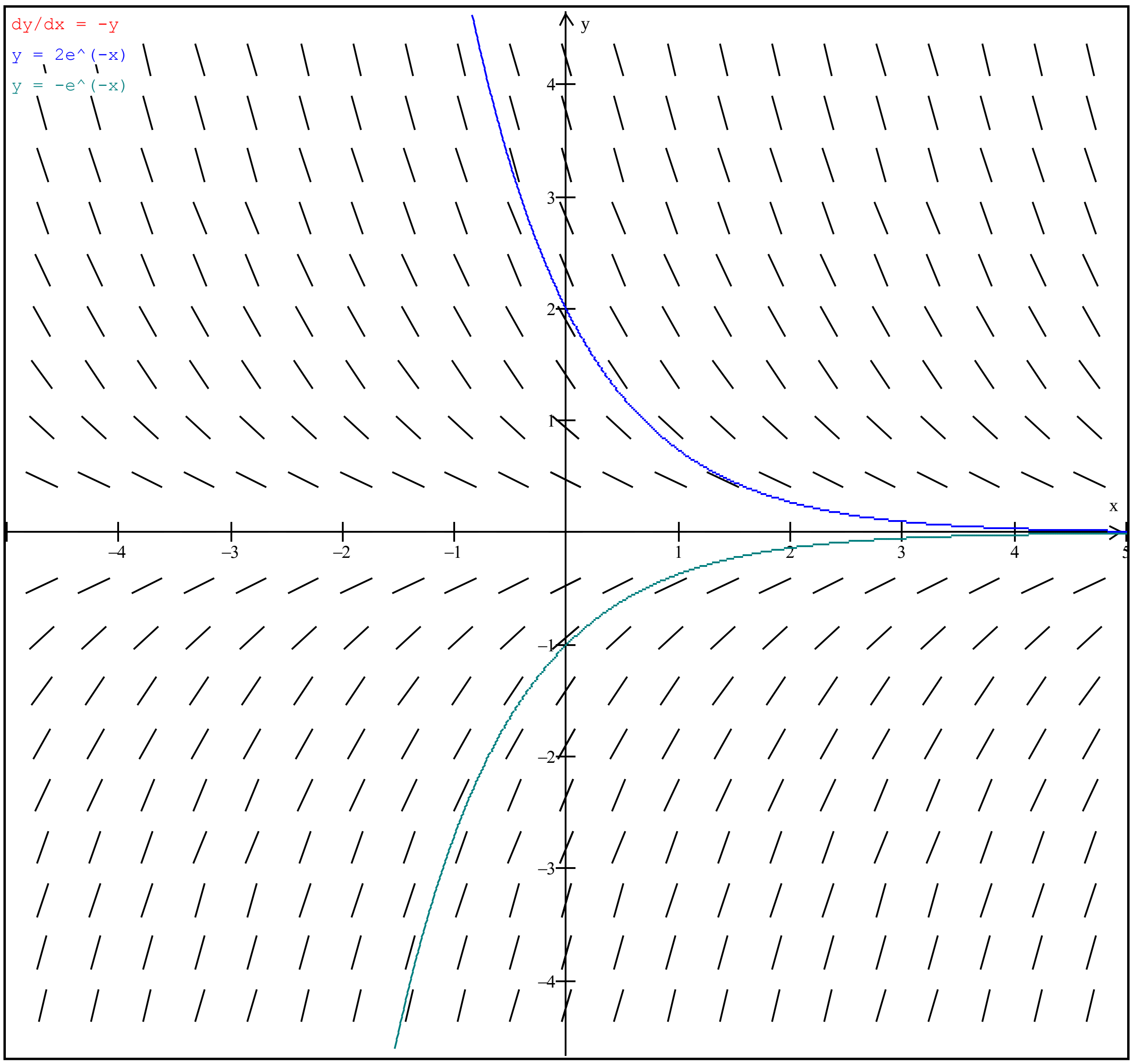
Group 6: Mass Moving on a Plane with Friction.

$$M\frac{dv}{dt} = -Bv \quad ; \quad v(t=0) = v_0$$

M – known mass ; B – Known friction coefficient ; $v(t)$ – unknown function to be determined.

Slope field for M=1, B=1, : $1 * \frac{dv}{dt} = -v$

Solution curves: Initial condition v(0) = 2 , and v(0)=-1.



Solve for general M,B,v(0).

What does exponential decay mean here? Half speed time?

Little helper

Group 7: Population Growth Models. (H)

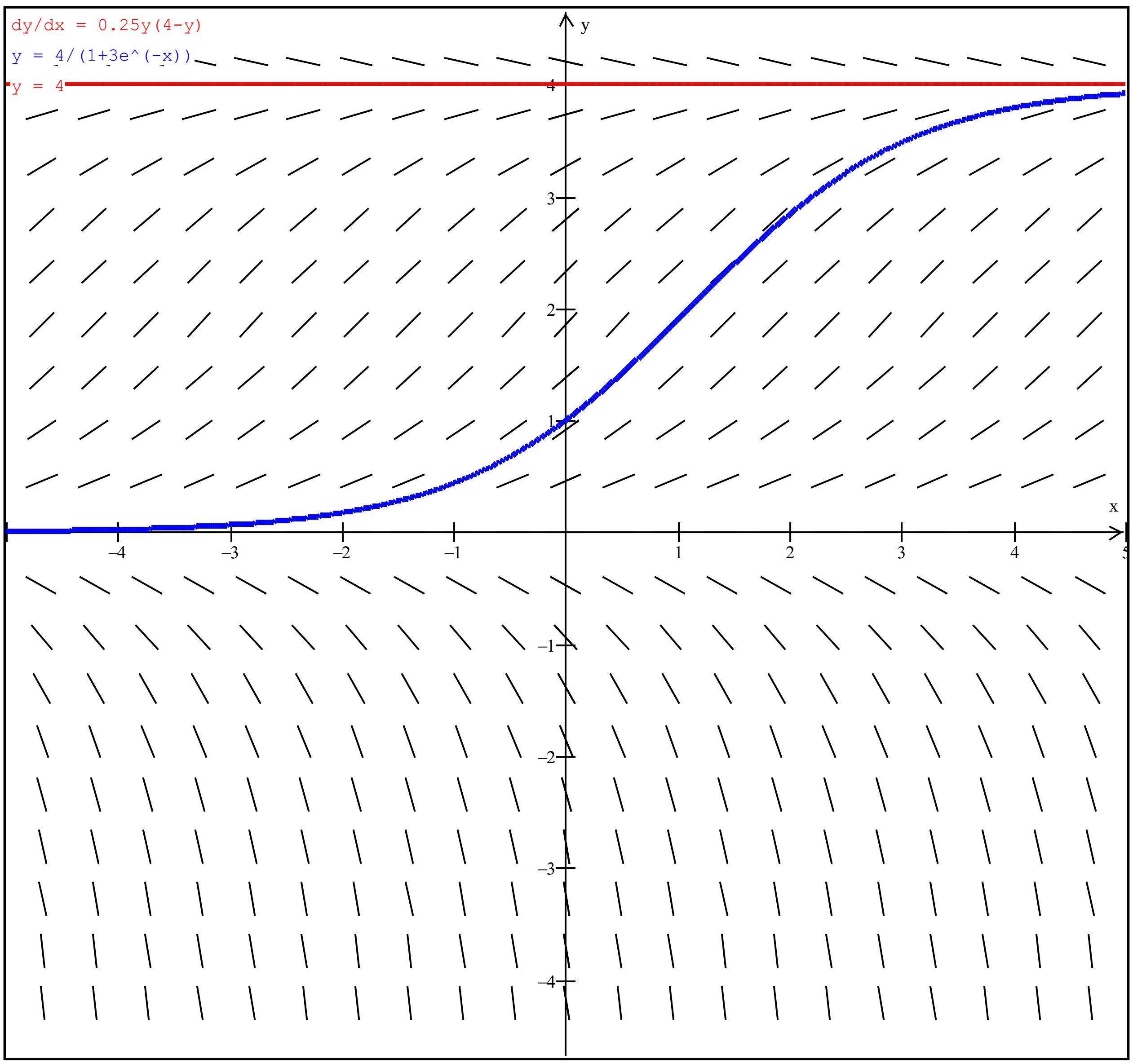
$$\frac{dN}{dt} = kN(N_{equi} - N) \qquad ; \qquad N(t=0) = N_0$$

$N(t)$ is the population number, to be solved for. K, N_{equi} , and N_0 are known constants.

(HINT: look Logistic Equation, page 427 in the book)

Slope field for k=0.25 , $N_{equi} = 4$, $N_0 = 1$: $\frac{dN}{dt} = 2N(4 - N)$

Solution curve



Solve for general k , N_{equi} , N_0.

Explain meaning of different parameters.

Little helper

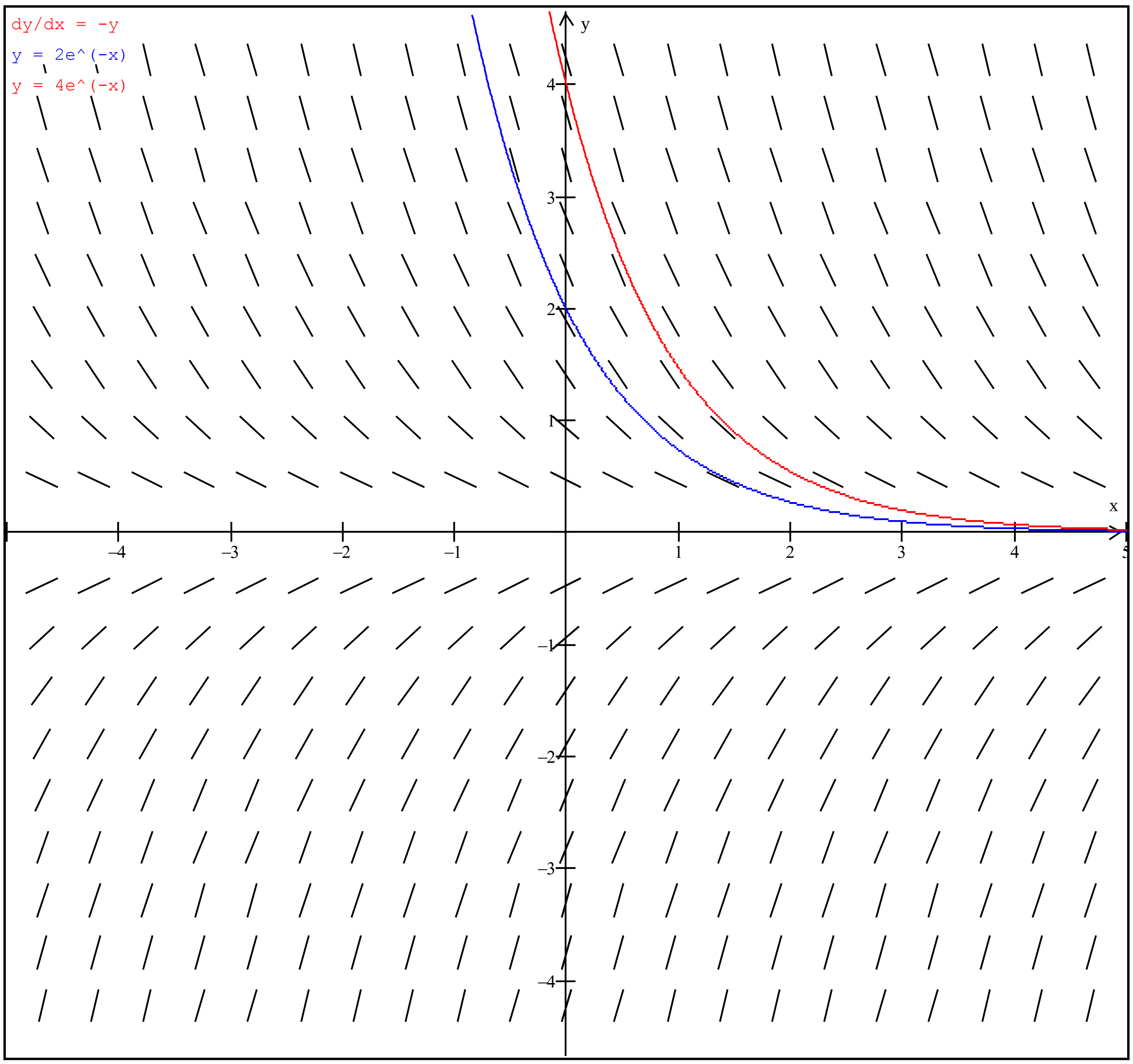
Group 8: Radioactive Decay.

$$\frac{dm}{dt} = -km \qquad ; \qquad m(t=0) = m_0$$

$m(t)$ is the mass to be solved for. k and m_0 are known constants.

Slope field for k=1 : $\frac{dm}{dt} = -1 * m$

Solution curves: Initial condition m(0) = 2 , and m(0)=4.



Solve for general k and m_0 .

Explain exponential decay, 'half-life'.