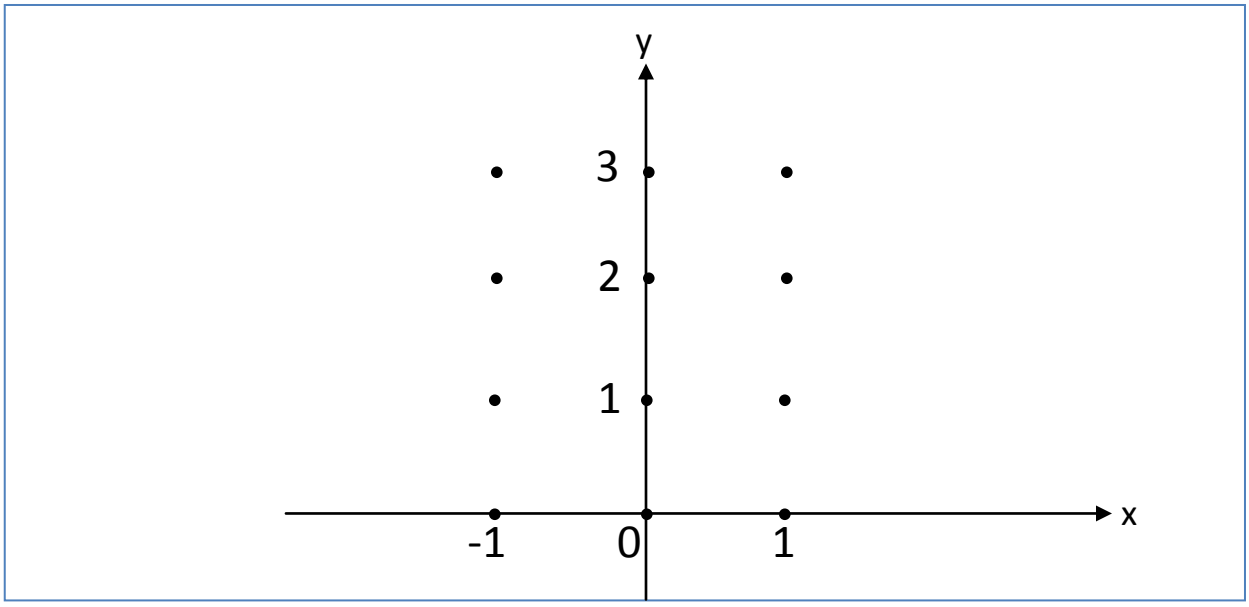
Student Name:_____ Date:_____

Differential Equations: Unit test (Version A)

Question 1 (30 pts) (Based on AP-2004)

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) While the slope field in part (a) is drawn at only dozen points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are non-negative.

(c) Find the particular solution y=f(x) to the given differential equation with the initial condition f(0)=3.

Question 2 (30 pts)

A cylinder of base area $A = 5\ in^2$ is filled from the top by a hose at a rate of $0.5\ (\frac{in^3}{minute})$. At the same time, it is leaking through a hole at the bottom at a rate of $Ay\ (\frac{in^3}{minute})$, where y is the level of water in the cylinder. Denote as V the volume of water in the cylinder .

(a) Draw a picture describing the problem.

(b) We can express the volume of the cylinder as $V = Ay$. Prove that: $A\frac{dy}{dt} = 0.5 - Ay$

(c) Find the general solution for the equation in (b). (← **EVEN if you didn't solve previous parts, start from here!!**)

(d) Find a particular solution for which at time t=0, the water are $1\ in$ high.

(e) What would be the level of the water as $\rightarrow \infty$?

Following are 4 'shorter' questions (40 pts overall)

Question 3 (10 pts)

4.1 $4y'' + 5 = 3x$ is a differential equation of order:

(a) 4 (b) 3 (c) 2 (d) 1

Explain:_____

4.2 $4y'' + 5 = 3x^2$ is a non linear differential equation in y:

(a) True (b) False

Explain:_____

4.3 The differential equation $y' = x + y$ can be solved by using Separation of Variables method:

(a) True (b) False

Explain:_____

Question 4 (10 pts)

Given the equation : $y' = 2^2 + y^2$, the general solution is (**Show your work!**):

a) $y = \frac{1}{2} arctan\left(\frac{x}{2}\right) + C$

b) $y = \frac{1}{2} arctan\left(\frac{x}{2} + C\right)$

c) $y = 2tan(2x) + C$

d) $y = 2tan(2x + C)$

e) None of the above.

Recall:

$$\frac{d}{dx}\arctan(u) = \frac{u'}{1+u^2}$$

Question 5 (10 pts)

(a) Given the differential equation $y' = 3x^2 + 2x - 5$, find the tangent line to the particular solution that passes through the point $(2, 3)$. .

Circle the right answer and explain (briefly):

(b) Let $y_1' = Q(x)$ and $y_2' = P(x)$, and assume all functions are defined for all real x. Also, assume Q(x)>P(x) . Is it true that for all real x, all particular solutions satisfy $y_1(x) > y_2(x)$?

True False

Explain:_____

_____

Question 6 (10 pts) (Based on AP Question)

If $\frac{dy}{dx} = y\,sec^2 x$, and y=5 when x=0 , then y = (**Show your work!**)

(a) $e^{tan\,(x)} + 4$

(b) $e^{tan\,(x)} + 5$

(c) $5e^{tan\,(x)}$

(d) $tan(x) + 5$

(e) $tan(x) + 5e^x$

Recall:

$$\int \sec^2(u)du = \tan(u) + C$$

----------===End===----------

Reflections: (Priceless):

The test was : Very-Easy 1 2 3 4 5 6 7 8 9 10 Very-Hard .

I think I got (0 to 100) : _____

How well was I prepared to the test: Not-prepared 1 2 3 4 5 6 7 8 9 10 Very-well prepared

I f I need to prepare again, I would (do differently): _____

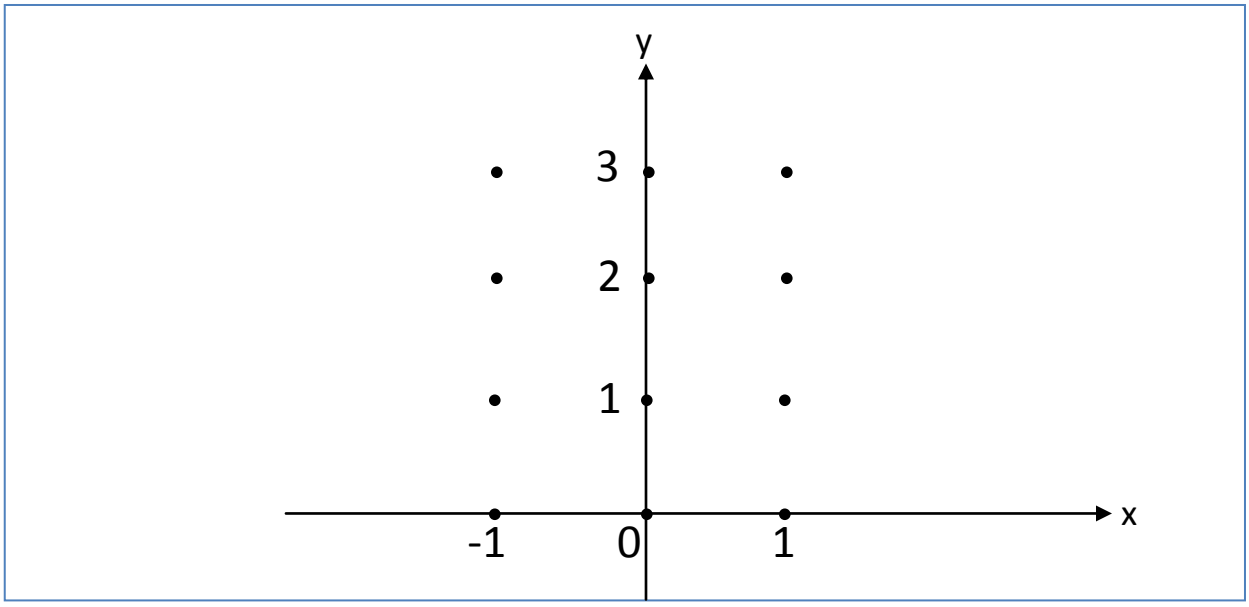
Student Name:_____ Date:_____

Differential Equations: Unit test (Version B)

Question 1 (30 pts) (Based on AP-2004)

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) While the slope field in part (a) is drawn at only dozen points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution y=f(x) to the given differential equation with the initial condition f(0)=4.

Question 2 (30 pts)

A cylinder of base area $A = 4\ in^2$ is filled from the top by a hose at a rate of $0.4\ (\frac{in^3}{minute})$. At the same time, it is leaking through a hole at the bottom at a rate of $Ay\ (\frac{in^3}{minute})$, where y is the level of water in the cylinder. Denote as V the volume of water in the cylinder .

(a) Draw a picture describing the problem.

(b) We can express the volume of the cylinder as $V = Ay$. Prove that: $A\frac{dy}{dt} = 0.4 - Ay$

(c) Find the general solution for the equation in (b). (← **EVEN if you didn't solve previous parts, start from here!!**)

(d) Find a particular solution for which at time t=0, the water are $1\ in$ high.

(e) What would be the level of the water as $\rightarrow \infty$?

Following are 4 'shorter' questions (40 pts overall)

Question 3 (10 pts)

4.1 $4y'' + 5 = 3x$ is a differential equation of order:

(a) 4 (b) 3 (c) 2 (d) 1

Explain:_____

4.2 $4y'' + 5 = 3x^2$ is a non linear differential equation in y:

(a) True (b) False

Explain:_____

4.3 The differential equation $y' = x + y$ can be solved by using Separation of Variables method:

(a) True (b) False

Explain:_____

Question 4 (10 pts)

Given the equation : $y' = 3^2 + y^2$, the general solution is (**Show your work!**):

a) $y = \frac{1}{3} arctan\left(\frac{x}{3}\right) + C$

b) $y = \frac{1}{3} arctan\left(\frac{x}{3} + C\right)$

c) $y = 3tan(3x) + C$

d) $y = 3tan(3x + C)$

e) None of the above.

Recall:

$$\frac{d}{dx}\arctan(u) = \frac{u'}{1+u^2}$$

Question 5 (10 pts)

(a) Given the differential equation $y' = 3x^2 + 2x - 5$, find the tangent line to the particular solution that passes through the point $(2, 3)$. .

Circle the right answer and explain (briefly):

(b) Let $y_1' = Q(x)$ and $y_2' = P(x)$, and assume all functions are defined for all real x. Also, assume Q(x)>P(x) . Is it true that for all real x, all particular solutions satisfy $y_1(x) > y_2(x)$?

True False

Explain:_____

_____

Question 6 (10 pts) (Based on AP Question)

If $\frac{dy}{dx} = y\,sec^2 x$, and y=4 when x=0 , then y = (**Show your work!**)

(a) $e^{tan\,(x)} + 4$

(b) $e^{tan\,(x)} + 5$

(c) $4e^{tan\,(x)}$

(d) $tan(x) + 4$

(e) $tan(x) + 4e^x$

Recall:

$$\int \sec^2(u)du = \tan(u) + C$$

----------===End===----------

Reflections: (Priceless):

The test was : Very-Easy 1 2 3 4 5 6 7 8 9 10 Very-Hard .

I think I got (0 to 100) : _____

How well was I prepared to the test: Not-prepared 1 2 3 4 5 6 7 8 9 10 Very-well prepared

I f I need to prepare again, I would (do differently): _____