Unit Plan: Differential Equations

Lesson #	Subject	Details	Special comments / HW
1	* Introduction.	*Connection to Prior-knowledge. * Motivation. * Map of the unit + description. * Terms we will use. * Verifying solution.	P. 409 : 1, 2, 4, 5, 8, 13, 14, 15
2	* General solution and verifying solution. * Initial conditions and Particular solution.	* Use simple cases students already know how to solve. * Important: The solution is a FUNCTION, not a number!	P. 409 : 31,32,36 P. 410 : 37, 43,48
3	* Solution curves + Slope fields.	* Start plotting on transparencies.	
4	* Slope fields (cont.)	* This is one of the critical subjects, so worth spending another full lesson on!	
5	* Unit project.	* This can be moved around. * The reason to put it here initially is to fit on 'Casual Friday'.	**Collect homework** (for the whole week).
6	* Euler method.	* Numerical solution.	
7	* Separation of Variables.	* The only method tested. * Exponential solutions – Most important ones!	
8	* Exponential solutions + Logistic equation.	* Graphing organizer of the unit.	
9	* Unit Project.	* Some more work and examples. * Work on unit-project: Applications!! And meaning of solutions.	Write three question (+solutions) for the test: Easy, Medium, Hard.
10	* Unit project.	* Finishing it all up. (also, working in class on the 3-questions)	**Collect homework** (for the whole week).
11	* (Optional) Finding orthogonal trajectories.	* This is a good application to explore, and brings some insight.	
12	Presentation of unit-projects.	* Peers evaluation.	
13	Unit-Review.	* Summary sheet.	
14	Test.		

*** Homework assignments** will be given at the end of each lesson, and updated on the web-site, according to our progress.

* Lesson plans, handouts, and additional information will be added to the web-site as we go along. (Though everything will be handed in class as well).

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Some general notes:

Top Down considerations: "External constraints"

1. About 2.5 weeks for the whole unit (not less than 2 weeks, and no more than 3 weeks).
2. Cover Chapter 6 in the book: "Differential Equations". This covers material needed for the AP exam.

Bottom up considerations: Things we view as focal points to the subject.

NOTE: According to the curriculum required for the AP Calculus AB and BC: (See Appendix A)

AB:
• Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

BC adds:
+ Numerical solution of differential equations using Euler's method

Below is the material in the chapter, and I will comment on which I think is relevant in light of the above:

1. Introduction: ← Needed. This is plenty, and some is really important, and will take a few lessons to cover.
    a. General solution.
    b. Verifying solution
    c. Characteristic curves.
    d. Initial conditions and Particular solution.
2. Slope fields. ← Needed for AB!
3. Euler's method – numerical approximation of solution. ←Gain insights! Needed for BC.
4. Separation of variables: ← Useful to go over just because they use it all the time.
    a. Exponential solution: Growth and Decay. ← important example/application
        i. Constant as a multiplier rather than addition.
        ii. Make sure students realize that there are equivalent forms: $e^{-at} = (e^{-a})^t$.
    b. Logistic equation. ← important example/application
    c. Good application to explore: Finding orthogonal trajectories.
5. Homogeneous equation. ← Skip
6. First order linear differential equation. ← Skip
7. Bernoulli equation. ← Skip

Lateral considerations: A project to span the whole unit.

We will have a unit-project that will span the whole unit period. Will start in the second lesson, and will be finished and presented by the end of the unit. It will be done in groups. Description of the project is in a separate document.

Calendar

February 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16 **Week off**	17 **In IHS**	18 ←	19 ←	20 ←	21	22
23	24	25	26	27	28	

March 2009

Mon	Tue	Wed	Thu	Fri	Sat	Sun
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

Appendix A : Calculus AB and BC requirements.

Curriculum needed for the AP exam (and some details about the exams):

Taken verbatim from :
ap08_calculus_coursedesc.pdf :: http://www.collegeboard.com/student/testing/ap/calculus_ab/samp.html

===== Start of "Verbatim" from AP document===

Topic Outline for Calculus AB

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Teachers may fi nd that topics are best taught in different orders. (See AP Central [apcentral.collegeboard.com] for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

II. Derivatives

Concept of the derivative

• Derivative presented graphically, numerically, and analytically
• Derivative interpreted as an instantaneous rate of change
• Derivative defined as the limit of the difference quotient
• Relationship between differentiability and continuity

Derivative at a point

• Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
• Tangent line to a curve at a point and local linear approximation
• Instantaneous rate of change as the limit of average rate of change
• Approximate rate of change from graphs and tables of values

Derivative as a function

• Corresponding characteristics of graphs of f and $f\_$
• Relationship between the increasing and decreasing behavior of f and the sign of $f\_$
• The Mean Value Theorem and its geometric interpretation
• Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

• Corresponding characteristics of the graphs of f, $f\_$, and $f\_$
• Relationship between the concavity of f and the sign of $f\_$
• Points of inflection as places where concavity changes

Applications of derivatives

• Analysis of curves, including the notions of monotonicity and concavity
• Optimization, both absolute (global) and relative (local) extrema
• Modeling rates of change, including related rates problems
• Use of implicit differentiation to find the derivative of an inverse function
• Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
• Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

Computation of derivatives

• Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
• Derivative rules for sums, products, and quotients of functions
• Chain rule and implicit differentiation

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Topic Outline for Calculus BC

The topic outline for Calculus BC includes all Calculus AB topics. Additional topics are found in paragraphs that are marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fi t immediately after their Calculus AB counterparts. Other topics may fi t best after the completion of the Calculus AB topic outline. (See AP Central for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I just went to the relevant spot here, and highlighted part:

Applications of derivatives

• Analysis of curves, including the notions of monotonicity and concavity
+ Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration
• Optimization, both absolute (global) and relative (local) extrema
• Modeling rates of change, including related rates problems
• Use of implicit differentiation to find the derivative of an inverse function
• Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
• Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
+ Numerical solution of differential equations using Euler's method
+ L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series

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Later on:

T H E E X A MS

The Calculus AB and BC Exams seek to assess how well a student has mastered the concepts and techniques of the subject matter of the corresponding courses. Each exam consists of two sections, as described below.
Section I: a multiple-choice section testing profi ciency in a wide variety of topics
Section II: a free-response section requiring the student to demonstrate the ability to solve problems involving a more extended chain of reasoning
The time allotted for each AP Calculus Exam is 3 hours and 15 minutes. The multiple-choice section of each exam consists of 45 questions in 105 minutes. Part A of the multiple-choice section (28 questions in 55 minutes) does not allow the use of a calculator. Part B of the multiple-choice section (17 questions in 50 minutes) contains some questions for which a graphing calculator is required.
The free-response section of each exam has two parts: one part requiring graphing calculators, and a second part not allowing graphing calculators. The AP Exams are designed to accurately assess student mastery of both the concepts and techniques of calculus. The two-part format for the free-response section provides greater fl exibility in the types of problems that can be given while ensuring fairness to all students taking the exam, regardless of the graphing calculator used.
The free-response section of each exam consists of 6 problems in 90 minutes. Part A of the free-response section (3 problems in 45 minutes) contains some problems or parts of problems for which a graphing calculator is required. Part B of the freeresponse section (3 problems in 45 minutes) does not allow the use of a calculator.
During the second timed portion of the free-response section (Part B), students are permitted to continue work on problems in Part A, but they are not permitted to use a

calculator during this time.
In determining the grade for each exam, the scores for Section I and Section II are given equal weight. Since the exams are designed for full coverage of the subject matter, it is not expected that all students will be able to answer all the questions.

Calculus AB: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus AB are included in the following sections. Answers to the sample questions are given on page 27.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the exam.

Part A consists of 28 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I, Part A, and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x$ = $\arcsin x$).

Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I, Part B, and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fi ll in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

===== End of "Verbatim" from AP document===

The following page is the back-side of the unit-plan page that will be given to the students.

Student Name: _____ Date:_____

Unit Summary worksheet