

Birthday Problem (lesson plan)

Overview

This activity uses the Birthday Problem as a motivation to introduce the concept of complementation for solving probability questions. Once faced with the problem, the students might be able to find a solution for a small-number of participants using their prior knowledge, but it is not practical to extend it to larger numbers of participants. Moreover, once we develop analytically the solution, the final result is very counterintuitive. The students will then verify this result with a few experiments.

Learning Objectives

Students will understand and be able to use the complementation rule in probability computation.

Prior Knowledge needed

Know the notion of independent events, and can use the rules for addition and multiplication to solve for probabilities of particular events in finite sample spaces.

Instruction and activity

1. Prior-Knowledge review/reminder: Let the students solve a simple probability question requiring the use of addition and multiplication rules. For example:
 - a. Rolling two dice, and requiring the sum of the resulting numbers to be 7 (Addition of events).
 - b. Pulling out balls from a collection of 7-red and 7-blue, no return, and requiring to get 2-reds and 1-blue in this specific order (Multiplication of events).
2. Pose the following question, and collect 'intuitive' answers :

What is the chance that in a group of 23 people we'll have two people (or more!) with the same Birth-date?

In group of 46 people?

In group of 92 people?

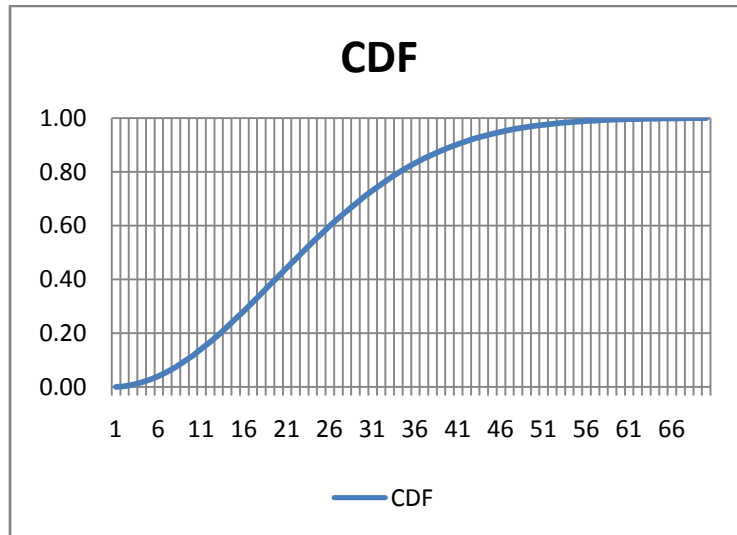
In group of 184 people?

In group of 365 people?

3. Guide them through a solution using addition and multiplication of events only.
 - a. Ask them, in groups of 2, to solve the problem for the case of 2-people. Compare results between all.
 - b. Ask them, in groups of 2, to solve the problem for the case of 3-people.
 - c. At this stage, when comparing the results for 3-people in the large forum, there will be many groups who made mistakes. It will be useful to re-align all, and talk together how it will look for 4-people, 5-people, and so forth.
 - d. The resulting solution is VERY cumbersome, and does not lend itself to large numbers.

4. Introduce the idea of complementation:
 - a. Mention similarities to other related cases: Logic - double negative like “Not-Not”; Math - multiplying negative numbers.
 - b. Solve the problem using complementation.

5. Calculate the resulting function, and solve the problem numerically.
 - a. Contrast the result with their ‘intuitive’ guesses.
 - b. Confirm the results with the ones they got ‘by hand’ for the 2 and 3 people cases.



6. Experiment in class (Each student can choose more than one b-day).

7. (optional) Calculator simulation: use the 1:365 notation for Birthday.

8. Wrap-up : Remind students of the place of complementation in their bag-of-tools to deal with probability problems.

9. Homework: Questions requiring complementation, and a few that show one can reach the same solution using different approaches!

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